

Midterm - Probability I (2026-27)

Time: 3 hours.

Attempt all questions. The maximum you can score is 30.

NOTE: You may quote any result proved in class without proof.

1. There are 5 hotels in a certain town. Suppose 3 people check into hotels of that town on September 8, each of them choosing a hotel at random. What is the probability that they each check into a different hotel? **[4 marks]**
2. A box contains 10 black balls and 5 red balls. At each turn, a ball is drawn at random from the box. It is then put back in the box, and *in addition* 4 balls of the same colour and 6 balls of the other colour are put into the box. Suppose we do this for 3 turns. What is the probability that all three balls selected are black? **[4 marks]**
3. If 8 people (consisting of 4 married couples) are randomly arranged in a row, find the probability that no husband sits next to his wife. **[5 marks]**
4. A gambler has a fair coin and another coin with heads on both sides. He selects one of the coins at random; when he flips it, it shows heads. What is the probability that it is the fair coin? **[4 marks]**
5. A sample space has 12 equally likely outcomes. Construct events A and B such that

$$P(A) = \frac{1}{2}, \quad P(B) = \frac{1}{3},$$

and A and B are independent. **[4 marks]**

6. For events $A_1, A_2, \dots, A_n$ in a sample space show that

$$P(\cap_{i=1}^n A_i) \geq 1 - \sum_{i=1}^n P(A_i^c). \quad \mathbf{[5 \text{ marks}]}$$

7. Consider a coin with probability of heads equal to p and probability of tails equal to $1 - p$. Let P_n be the probability that there are an even number of heads in n tosses. Determine P_n for each n . **[5 marks]**